AUTOMATIC INCORPORATION OF NUMERICAL SHORT-RANGE PROGNOSES IN 5-DAY CIRCULATION FORECASTS

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ABSTRACT

One of the basic aids in the preparation of extended forecasts is a 5-day mean mid-tropospheric chart centered on the day of the forecast, with associated instantaneous height tendencies, the unknown data being computed by autocorrelation techniques. Substitution of values from the barotropic 24-hour and 48-hour numerical prognoses for these unknowns objectively incorporated recent gains in short-period prognoses. Comparison of the statistically and physically computed mean charts and their tendencies suggests superiority of the latter over North America and the Atlantic, but not over the Pacific where errors in the barotropic prognoses, which are probably due to improper boundary assumptions and omitted physical processes, play an important role. Dynamic considerations can be applied to the new mean chart to suggest subsequent changes which may not be indicated kinematically.

1. INTRODUCTION

Within a few months after the beginning of routine numerical forecasting in May 1955, the Joint Numerical Weather Prediction (JNWP) Unit was producing a number of short-range prognoses derived from both barotropic and baroclinic prediction models. On receipt of these prognostic charts extended-range forecasters were confronted with a wealth of new information posing problems as to how to use it in practice. Several questions arose: (1) What was the prevailing accuracy of the various forecasts, both on an absolute scale and relative to each other, and to subjectively produced short-range prognoses? (2) If skillful, how could the prognoses best be assimilated in the 5-day forecast procedure?

Verification soon demonstrated that the currently available numerical prognostic charts had definite skill for 24 to 48 hours, but little skill beyond that time. For this reason, they obviously could find little use in directly estimating the circulation for a 5-day period commencing some 36 hours in the future. However, the U. S. Weather Bureau's procedure in extended forecasting has long included the use of a mean map centered on forecast day (called a regression chart or half-week mean map), together with its centered mean height tendencies, as a stepping-stone in getting to the circulation pattern of the forecast period [1]. The constant-pressure height data for approximately the second half of this half-week period are unknown and therefore must be estimated in some way. In standard operating procedure these unknown heights have been estimated statistically by a simple linear regression equation based on the auto-lag-correlation of constant pressure heights at each standard latitude-longitude intersection [1]. Although this is fairly satisfactory for representing the contours and centered 2-day height tendencies of this mean chart, it was felt that a dynamical prognosis of the unknown height data might prove superior to a crude statistical technique. This seemed to be the very place where the new numerical prognoses could potentially aid most in the current 5-day forecasting procedure—that is, by providing a sounder estimate of the current state of the large-scale circulation and its immediate changes. The experiment described here is a first attempt at evaluating and using the JNWP numerical prognoses objectively in the 5-day forecast routine.

2. THE PROBLEM OF COMPUTING THE HALF-WEEK CHART

The half-week chart is the mean of the daily upper-air charts for five consecutive days, centered on forecast day, with a centered mean tendency superimposed. In this experiment only one chart per day was utilized, so that the mean height at each standard intersection of latitude and longitude is given by

$$\overline{Z}_0=(1/5)[Z_{-2}+Z_{-1}+Z_0+Z_{+1}+Z_{+2}] \qquad (1)$$

where $\overline{Z}_0$ is the half-week mean chart value and the subscripts refer to maps for each day with $+1$ and $+2$ referring to 24 and 48 hours in the future, etc.

These last two values, Z_{+1} and Z_{+2}, are unknown and must be estimated in some way.

A. STATISTICAL METHOD

Because of the rather high lag correlation of constant-pressure heights, the departure from the monthly normal of the latest observed value of height (Z_0) correlates positively with the departure from the normal of the sum of the subsequent two unknown heights ($Z_{+1}+Z_{+2}$). This

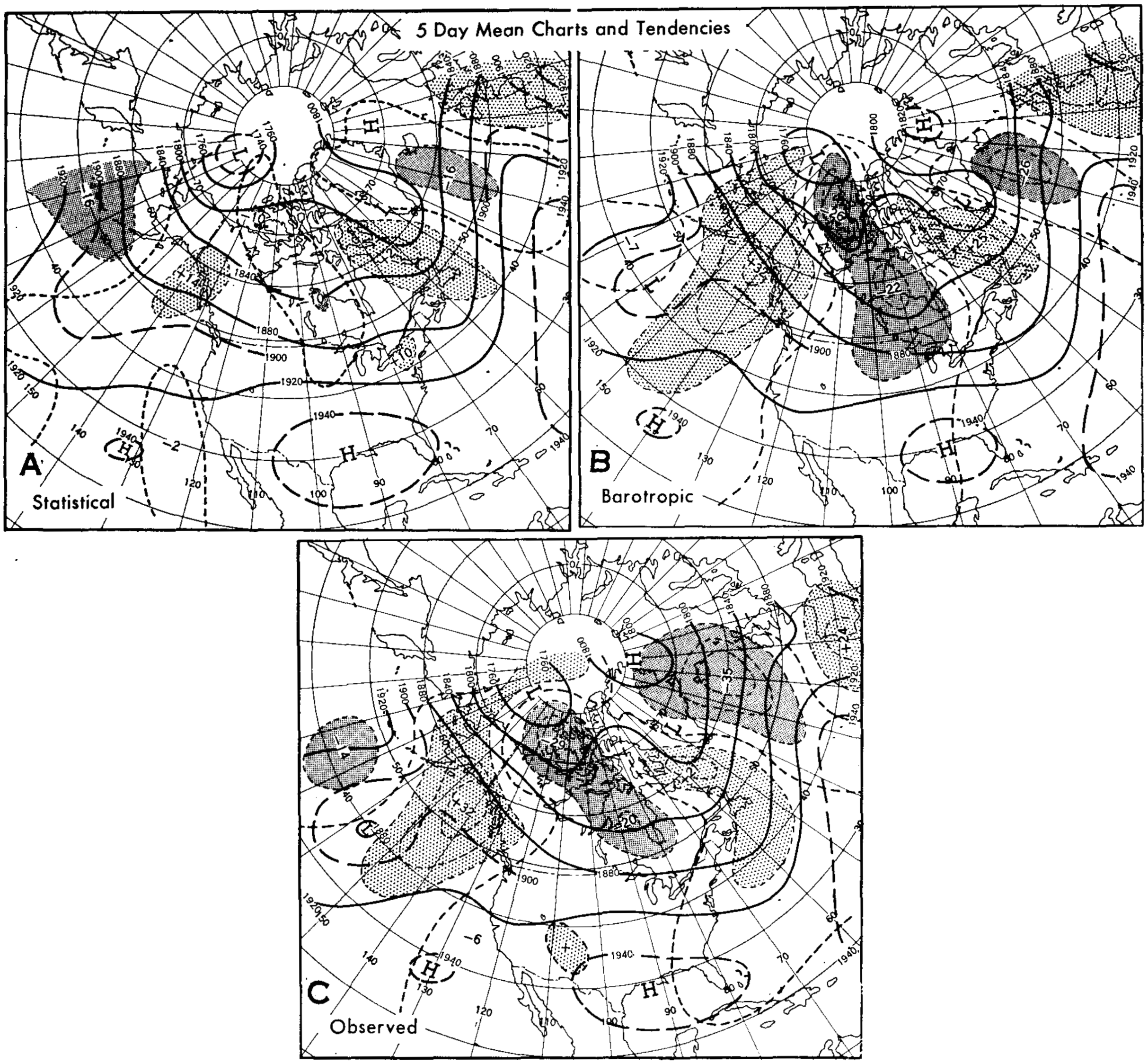

FIGURE 1.—Three half-week charts for August 7–11, 1956—(A) the statistical and (B) the barotropic estimates, and (C) the verifying observed chart. Solid lines are 5-day mean 500-mb. contours (in tens of feet) and dashed lines are 2-day mean height tendencies (in tens of feet). Tendency values greater than 100 feet per 2 days are shaded (dark for negative, light for positive). Barotropic method excelled in estimating the tendencies, especially the magnitude.

correlation is sufficiently large to make possible the use of a regression equation for estimating the two unknown values. The form of the equation is

$$(Z_{+1}-N)+(Z_{+2}-N)=K(Z_0-N) \qquad (2)$$

where N is the 500-mb. normal height at the point, and K is approximately equal to 1 at almost all points. Since in practice, at least for the present time, it is convenient to work with whole height values rather than fractions, K is set equal to 1 in equation (2). This gives

$$Z_{+1}+Z_{+2}=Z_0+N \qquad (3)$$

as the statistical estimate for the two unknowns. Substituting this into equation (1) we have

$$\overline{Z}_0=(1/5)[Z_{-2}+Z_{-1}+2Z_0+N] \qquad (4)$$

For a more complete discussion of this statistical method refer to pages 41 to 46 of [1].

Mean, centered, 2-day height tendencies are approximated by subtracting the 5-day mean height centered on the day before from that centered on the day after forecast day. Thus, the formula for the tendencies is:

$$T_0=(1/5)[Z_{+2}+Z_{+3}-Z_{-2}-Z_{-3}] \qquad (5)$$

Using the same procedure as employed in computing the mean height, the sum of the unknowns $(Z_{+2}+Z_{+3})$ is estimated using a regression equation, and again it turns out that (Z_0+N) is the best estimate for $(Z_{+2}+Z_{+3})$.

Figure 1A shows an example of a statistically computed half-week chart.

B. USE OF NUMERICAL FORECASTS

A *dynamically* computed half-week chart (fig. 1B) is obtained by utilizing the short-range numerical prognoses for approximating the unknown heights in equations (1) and (5). Ideally, these unknown daily height values should be related to the numerical prognoses by a multiple regression equation. However, because of the rather small samples of homogeneous data, resulting from frequent changes in the model used to prepare the numerical prognoses and because further changes in the models are contemplated, it is not practical to determine the multiple regression equation at this formulative stage of numerical prediction. For these reasons, as a temporary expedient the sum of two numerical prognoses was directly substituted for the sum of the two unknown heights. However, correlation coefficients (r) were computed between the sum of the two estimated 500-mb. heights, required to compute the half-week chart with its tendencies, and the sum of the corresponding two observed values, and furthermore, root mean square (RMS) and average errors were computed to determine the best combination of prognoses to substitute for the unknown. These dynamical estimates were also compared with the statistical approximations. This was done at a few selected locations for two periods during 1956 when relatively homogeneous samples were available. The results for the first period, April 20 to June 30, are shown in table 1. In order to decrease the time-dependence of individual observations in the sample, values were taken either 3 or 4 days apart, and since the thermotropic prognoses were computed only 4 days per week, the total sample contained only 21 cases. Because absolute height values, instead of departures from monthly normals, were employed as data the correlations should be used only to compare the various estimates at one location.

As indicated earlier, 500-mb. heights for 1 and 2 days in the future are needed for computing the mean contours (table 1A). The various estimates tested were: (1) the statistical forecast, which is simply the sum of the latest observed map (Z_0) and the appropriate monthly 500-mb. normal height (N); (2) the sum of the JNWP thermotropic [1] 24-hour and 36-hour prognoses [2]; and (3) the sum of the JNWP barotropic [2] 24-hour and 48-hour prognoses.

For this small sample there was little significant difference between the various methods of estimating the unknowns except in northwestern Canada where the physical prognoses were superior. In the Aleutians, the average error of the numerical prognoses was large and positive, indicative of a marked bias.

To compute the centered tendency, estimates of the daily heights 2 and 3 days in the future are needed (table 1B). The statistical estimate remains the same; i. e., the latest observed chart plus the normal. The barotropic 48-hour and 72-hour prognoses were available for these times, but corresponding thermotropic forecasts were not. Consequently, only the 36-hour thermotropic was used.

The correlations in the first three rows in section B of table 1 indicate that for this sample the substitution of numerical short-range prognoses is about as good as the use of the statistical technique. However, considerable deterioration of the numerical forecasts is evident in the Aleutians where the barotropic is definitely inferior to the statistical method. Perhaps the most surprising result is the superiority of the 36-hour thermotropic over the 48-hour and 72-hour barotropic in approximating the sum of the daily maps 2 and 3 days in the future. This may indicate, if the difference in models is neglected, that a rapid decay of the barotropic prognoses exists [3] and suggests that perhaps the 24-hour and 48-hour barotropic forecasts would be better approximations of the daily maps 2 and 3 days in the future than the corresponding 48-hour and 72-hour forecasts. The correlations tend to substantiate this hypothesis especially in the western area, as indicated by the bottom row (7) of table 1.

The second sample was for the summer season 1956 (June, July, and August) and the results are shown in

[1] The original model was quasi-geostrophic and included some mountain effects.

[2] The new model introduced April 20, 1956 was quasi-divergent and contained no mountain effects.

TABLE 1.—*Correlations (r) between the sums of the estimated 500-mb. heights, required to compute the half-week chart with its tendencies, and the sums of the corresponding two observed values; and also, the root mean square (RMS) and average errors (in tens of feet) of the sums of the two estimated values. These are for the period April 20 to June 30, 1956 at the indicated locations in North America. Z_n is 500-mb. height n days subsequent to forecast day (n=0 is forecast day). N is 500-mb. normal height. Period in hours for which prognosis was made is shown in parentheses.*

Estimates	Aleutians			Northwestern Canada			Arkansas			Southeastern Canada		
	r	RMS error	Average error	r	RMS error	Average error	r	RMS error	Average error	r	RMS error	Average error
	A. Unknowns in Estimated 5-Day Mean Height ($Z_{+1}+Z_{+2}$)											
(1) Statistical (Z_0+N)	.71	60	+10	.57	46	−5	.80	20	−6	.79	79	+29
(2) Thermotropic (24+36)	.77	78	+56	.86	28	+6	.79	25	−1	.79	56	+28
(3) Barotropic (24+48)	.67	89	+63	.86	40	−28	.78	30	−19	.86	38	−5
	B. Unknowns in Estimated Mean Tendencies ($Z_{+2}+Z_{+3}$)											
(4) Statistical (Z_0+N)	.54	73	+8	.43	55	−3	.69	25	−5	.73	55	−6
(5) Thermotropic (36)	.62	---	----	.74	---	----	.77	---	----	.75	---	----
(6) Barotropic (48+72)	.34	110	+69	.51	82	−59	.57	53	−28	.51	70	+20
(7) Barotropic (24+48)	.52	96	+60	.85	40	−26	.61	35	−17	.79	49	−3

TABLE 2.—*Correlations (r) between the sums of the estimated 500-mb. heights, required to compute the half-week chart with its tendencies, and the sums of the corresponding two observed values; and also, root mean square errors (in tens of feet) of the sums of the two estimated values. These are for the period June 1–August 31, 1956 at the indicated locations in North America. Z_n is 500-mb. height n days subsequent to forecast day (n=0 is forecast day). N is 500-mb. normal height. Period in hours for which prognosis was made is shown in parentheses*

Estimates	Aleutians			Northwestern Canada			Idaho			Arkansas			Southeastern Canada		
	r	RMS error	Average error	r	RMS error	Average error	r	RMS error	Average error	r	RMS error	Average error	r	RMS error	Average error
	A. Unknowns in Estimated 5-Day Mean Height ($Z_{+1}+Z_{+2}$)														
(1) Statistical (Z_0+N)	.81	58	−7	.79	30	−4	.76	30	+7	.67	18	+8	.39	50	+20
(2) Thermotropic (24+36)	.91	45	+20	.79	35	0	.74	36	+3	.70	24	−10	.91	32	+7
(3) Barotropic (24+48)	.89	57	+36	.83	39	−21	.76	37	−15	.72	21	−13	.72	35	+7
	B. Unknowns in Estimated Mean Tendencies ($Z_{+2}+Z_{+3}$)														
(4) Statistical (Z_0+N)	.75	64	−10	.65	36	−4	.56	34	+5	.45	21	+8	.04	63	+18
(5) Thermotropic (36)	.83	---	----	.63	---	----	.68	---	----	.47	---	----	.82	---	----
(6) Barotropic (48+72)	.70	71	+32	.65	73	−21	.74	39	−16	.53	34	−13	.59	46	+5
(7) Barotropic (24+48)	.82	63	+21	.71	47	−48	.57	47	−23	.48	26	−12	.46	50	+22

table 2. All data were homogeneous since no significant changes were made during this period in the numerical forecasting models. Thermotropic prognoses during this period were prepared daily, and height values were read off every other day, so that the sample contained 41 cases. The same estimates were tested as in the earlier sample but one additional location, Idaho, was included.

The results are quite similar to those of the April–June period. The physical prognoses are as good as the simple statistical approach and perhaps slightly better on an overall basis. Two tentative observations can be made: (1) The average and RMS errors at the Aleutian location are less than in the earlier sample, indicating that the boundary error of the short-range numerical model was less during this summer. Of course some decrease in error is expected in the summer season when the variability of 500-mb. height is less, but at this western location the decrease is much greater than at the other locations. (2) In estimating the daily heights 2 and 3 days in the future ($Z_{+2}+Z_{+3}$) the barotropic 24-hour and 48-hour prognoses are not in general superior to the 48-hour and 72-hour prognoses. This suggests that there is a slower decay of the barotropic forecasts in summer than in spring.

Since the numerical prognoses apparently can produce results at least as good as and perhaps in some areas superior to the statistical method, the decision was made to prepare half-week mean charts on a routine basis using the dynamic prognoses. The selection of the exact prognoses to use was not obvious. However, in addition to the results presented here other verification studies [3, 4, 5] indicated that the barotropic 72-hour forecasts had considerably less skill than the other prognoses. Thermotropic prognoses could not be used, since after June 30, 1956 they were prepared from the 1500 GMT map, and were not available at the proper time to be incorporated into the forecast routine. Therefore, in the routine preparation the two unknown daily values needed in the computation of both the 5-day mean height and its tendencies were estimated by utilizing the 24-hour and 48-hour barotropic prognoses.

C. PREPARATION OF BAROTROPIC HALF-WEEK CHART

The routine preparation of the chart is done on the IBM 704 computer and therefore is extremely easy and rapid. No collecting of data is required since all the 500-mb. heights are available on punch-cards. They are a by-product of the JNWP short-range prognoses. Both the mean height and tendency values are computed, printed on a rectangular grid, and analyzed by the electronic machine. This entire process is completed in only 5 minutes.

3. USEFULNESS OF BAROTROPIC HALF-WEEK CHART FOR 5-DAY FORECASTING

A. COMPARISON OF ESTIMATED CHARTS

Both the dynamical and statistical half-week charts are routinely prepared each forecast day, so that continuous subjective evaluation is possible.[3] Some of the typical advantages and shortcomings of the two approximations are illustrated in figure 1. This contains three half-week charts for August 7–11, 1956—the statistical estimate, the barotropic estimate, and the observed.

The height field was well approximated by both techniques; the maximum error of the statistical estimate was 280 feet, while that of the barotropic estimate was no more than 140 feet. These two patterns are quite similar and either one is accurate enough for immediate purposes. However, the barotropic does tend to catch more of the detail than the statistical chart. The latter method flattens the height field and much of the detail is lost. For example, in the Pacific the closed Low is absent and the ridge just off the west coast of the United States is too flat.

It is more difficult to get a good estimate of the tendencies. Not only are the required data farther into the future, but the tendency is the small difference between large height values, so that small percentage errors in the estimated heights can result in intolerable errors in the tendencies.

[3] The statistical chart is routinely prepared for the 700-mb. level using 2 maps per day.

A quick examination of this case reveals that the general pattern of the tendencies of both estimates is similar, and that they both resemble those of the observed. However, the average intensities of the barotropic tendencies are larger than those of the statistical and more closely approximate those of the observed. In general, the statistically estimated tendencies have several deficiencies. They tend to "freeze" the pattern that exists on forecast day. Extreme height anomalies are almost always returned toward the normal.

B. USEFULNESS OF HALF-WEEK CHART—NECESSITY FOR FURTHER PHYSICAL CONSIDERATION

In its present form there are two major shortcomings of the barotropically estimated half-week chart for 5-day forecasting:

(1) It covers only about half the hemisphere, and in addition the Pacific area verifies poorly. Good height tendencies in the Pacific would assist greatly in the preparation of an extended-range forecast over the United States. This shortcoming may soon be overcome. The results of the new model covering the entire Northern Hemisphere, which will be introduced by the JNWP Unit this fall (1957), will be studied carefully with a view toward determining the best statistical relationship between barotropic prognoses and unknown heights. A regression equation would automatically remove any bias that might exist in the prognoses. This hemispheric grid, which is expected to diminish boundary errors, should result in better numerical prognoses in the Pacific.

(2) The tendencies are relatively short-period and instantaneous, and even if they are accurate they may not indicate the longer-range changes necessary for making a 5-day forecast. Supplemented with subjective physical reasoning they may suggest the correct solution, but do not directly indicate it.

Two series of 5-day mean charts (figs. 2 and 3) will be used to illustrate two things: (1) that the instantaneous tendencies do not always indicate the longer-period changes; and (2) that it is sometimes, although not always, possible to determine when and how to modify the tendencies in order to make the correct full-week forecast. Both series of charts contain the initial observed chart, the barotropically estimated and the verifying observed half-week charts (with tendencies), and the verifying full-week chart with full-week change in height. Constant absolute vorticity (CAV) trajectories have been computed, using the observed half-week flow in the Pacific, to indicate in a rough manner the subsequent effects of vorticity flux from this upstream area on the flow pattern in the United States, or to determine if vorticity flux from the Pacific favored or opposed a continuation of the changes indicated by the instantaneous tendencies over the States. Obviously, the observed flow pattern would not be available on forecast day, but for this study it seemed advisable to base any arguments on the actual wind field rather than on an estimated one, especially since the Pacific region verifies rather poorly. In practice various other methods are used to determine this vorticity flux from the Pacific, but these will not be discussed in this report.

The first series (fig. 2) is an example in which the barotropically estimated tendencies, if considered as indicators of the full-week change, would lead to a good full-week forecast. If the half-week charts are compared (figs. 2C and D), it can be seen that both the contours and tendencies were quite accurately approximated. Furthermore, the 2-day estimated tendencies bear a remarkable resemblance to the full-week mean height change (fig. 2B). Also, the kinematic computation giving the instantaneous speed of the trough (not shown) suggests approximately the correct movement of the major trough in the United States.

These instantaneous trends worked well as full-week predictors because no major processes, such as vorticity advection, were operating to defeat them. CAV trajectories from the Pacific (fig. 2D) were in sympathy with the change in pattern that was indicated on "forecast day" over the United States.

The second series (fig. 3) is an example in which the half-week chart would lead to a poor full-week forecast over the United States if only the kinematic indications were used. As in the previous series, the half-week chart was well estimated. Admittedly, the tendencies were not perfect, but the positive tendencies in northwestern United States and small changes over the remaining area of the country were rather well approximated.

However, it can readily be seen that the instantaneous tendencies would lead to a poor forecast since large rises occurred in the West and marked falls in the East resulting in a 180° phase change of the westerly wave over the United States (fig. 3B). This rapid change, which was initiated after forecast day, can easily be explained by the flux of vorticity from upstream in the Pacific. The strong flow pattern in the eastern Pacific, which incidentally was supported by the westerly waves farther upstream, was not in sympathy with the half-week contour pattern in the United States. CAV trajectories supported a ridge over the western United States and a trough in the East where a sharp ridge existed and with no indication in the tendency field of any marked decrease in intensity. Therefore, instantaneous tendencies must be altered for full-week forecasting when they do not agree with a large flux of vorticity from upstream. Fortunately for this case, the flow pattern in the Pacific changed little subsequent to forecast day, so that the vorticity flux indicated on forecast day remained relatively unchanged throughout the prognostic period. In general, changes over the Pacific must be anticipated. This would be facilitated by good half-week tendencies in this area.

C. PRESENT LIMITATIONS AND EXPECTED IMPROVEMENTS

The barotropic are usually better than the statistical tendencies in the Atlantic and North America but inferior in the Pacific where large positive errors are observed. This is not true in the case shown in Figure 1 but is indicated in other examples. In this western region the short-

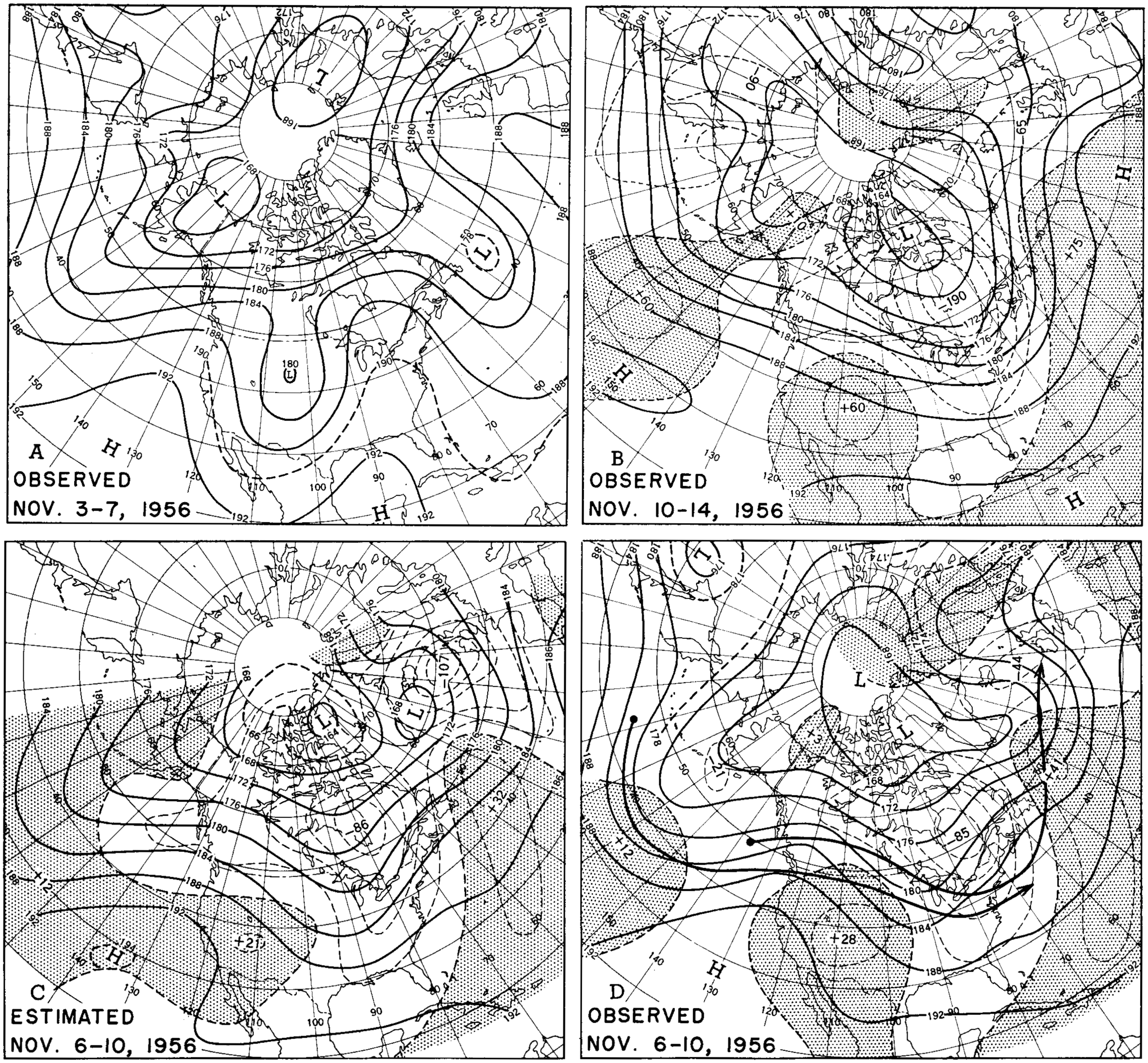

Figure 2.—5-day mean 500-mb. contours (solid lines, labeled in hundreds of feet) for periods indicated on charts. (A) Initial chart (observed) for November 3–7, 1956. (B) Verifying chart for November 10–14, 1956 with full-week height changes, Chart B minus Chart A, superimposed (dashed lines, 400 ft. interval, centers labeled in tens of feet, positive values stippled). (C) Half-week chart for November 6–10, 1956 with 2-day mean height tendencies (dashed lines, 200-ft. interval, centers labeled in tens of feet, positive values stippled) both of which are partially estimated by substitution of barotropic prognoses. (D) Observed (or verifying) half-week chart for November 6–10, 1956. Heavy solid arrows are spherical, constant absolute vorticity trajectories. Tendencies were well estimated, and they indicate correct full-week change.

range barotropic prognoses almost invariably contain large positive errors, especially in winter, which are probably due to unsuitable boundary assumptions in the model, baroclinicity, and non-adiabatic heating.

Attempts are now being made to develop methods for removing some of these errors, especially the bias, and for incorporating the improvement in the computational procedure of the half-week chart. First, the error fields were compared with prognostic height changes which were obtained by using the normal 500-mb. heights as initial data in the numerical forecasting procedure. Clapp [6] examined this method using normal maps, and more recently Williams [7], with the aid of an electronic computer, used both normal and mean charts. In

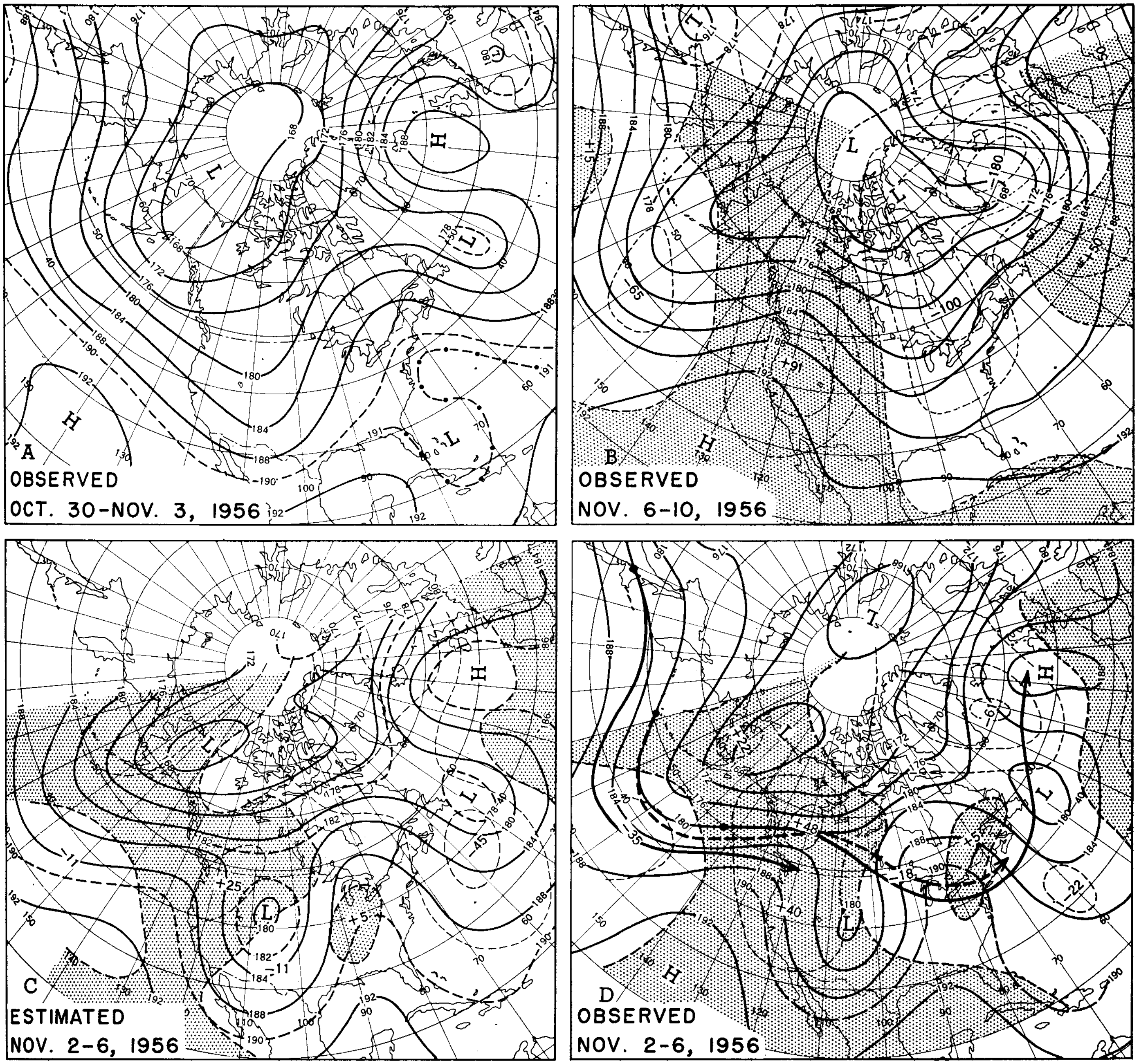

Figure 3.—5-day mean 500-mb contours for periods indicated on charts. See legend of figure 2 for details. The constant absolute vorticity trajectories have been drawn as heavy dashed or solid arrows. Tendencies are well estimated but do not indicate correct full-week change.

this brief investigation the prognostic height changes, which were obtained when the normal chart was used, correlated positively with the error fields although the coefficients were small.

In addition to the above work, programs for the electronic computer are being prepared and other ground work is being laid for computing multiple regression coefficients between the unknowns in the half-week chart and the various numerical prognostic heights. It is hoped this new dynamically estimated chart will prove superior on an overall basis and can replace the chart produced by the current autocorrelation method. An improved half-week chart will serve as a basis for numerical forecasting models, now being developed, for predicting time-averaged flow patterns[4] [8]. This means that the forecaster no longer would be forced to rely on outmoded vorticity trajectory methods to indicate the subsequent barotropic effects.

[4] Since this paper went to press these two programs have been incorporated into the routine forecasting procedure.

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